Maldacena-Nunez's no-go theorem for warped 11d de Sitter solution

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We start from the warped metric

$$\mathrm{d}s_{11}^2 = \Omega(y)^2 \left(g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \hat{g}_{mn}\mathrm{d}y^m \mathrm{d}y^n\right), \tag{1}$$

where μ and ν run from $0 \sim 3$, and m, n from 4 to 10. We will also use capital alphabets (Latin letters) L, M, N etc. to denote general 11-dimensional indices. The D-dim Ricci tensor in the 4-dim spacetime directions $R^{(D)}_{\mu\nu}$ can be written as

$$R^{(D)}_{\mu\nu} = R_{\mu\nu}(g) - g_{\mu\nu}\left[\widehat{\nabla}^2 \log\Omega + (D-2)(\widehat{\nabla}\log\Omega)^2\right], \tag{2}$$

where $\widehat{\Box}$ and $\widehat{\nabla}$ denote the derivatives in the compact 7-dim directions. From the e.o.m., we have

$$R^{(D)}_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-2}\Omega^2 g_{\mu\nu} T^L{}_L. \tag{3}$$

Note that

$$\widehat{\nabla}^2 \log\Omega + (D-2)(\widehat{\nabla}\log\Omega)^2 = \frac{1}{(D-2)\Omega^{D-2}}\nabla^2\Omega^{D-2}, \tag{4}$$

by contracting the e.o.m. with $g^{\mu\nu}$, we obtain

$$\frac{4}{(D-2)\Omega^{D-2}}\nabla^2\Omega^{D-2} = R(g) - \Omega^2\left(T^\mu{}_\mu - \frac{4}{D-2}T^L{}_L\right). \tag{5}$$

If $\tilde{T} := -T^\mu{}_\mu + \frac{4}{D-2}T^L{}_L \geq 0$, then we see that the assumption that we have a de Sitter space, with $R(g) > 0$, implies

$$\int \mathrm{d}^{D-4}y\ \sqrt{\hat{g}}\Omega^{D-2}\widehat{\nabla}^2\Omega^{D-2} > 0, \tag{6}$$

while the l.h.s. of the above can be integrated by part to

$$-\int \mathrm{d}^{D-4}y\ \sqrt{\hat{g}}\left(\widehat{\nabla}\Omega^{D-2}\right)^2, \tag{7}$$

which is apparently non-positive. This leads to the inconsistency. Maldacena-Nunez checked that $\tilde{T} \geq 0$ holds for 11d SUGRA with negative vacuum scalar potential (Why not positive? To have 11d AdS vacuum?) and 10d type IIA massive SUGRA.

For the scalar potential V, the corresponding stress tensor is

$$T_{MN} = -V g_{MN} \Rightarrow \tilde{T} = -V\frac{8}{D-2}, \tag{8}$$

and for negative potential, it is positive. For n-form field strength,

$$T_{MN} = F_{ML_1 \ldots L_{n-1}} F_N{}^{L_1 \ldots L_{n-1}} - \frac{1}{2n} g_{MN} F_{L_1 \ldots L_n} F^{L_1 \ldots L_n}, \tag{9}$$

Since we are considering the cosmological application, there are only two scenarios we need to discuss. One is that only $F_{mn \ldots l}$ components are non-zero, and the other is that only $F_{012 \ldots 3m \ldots n}$'s are non-vanishing, to preserve the isometry of $\mathrm{d}S^{1,3}$. We can also consider a mixed situation, but it is enough to discuss them separately.

In the first case,

$$\tilde{T} = -F_{\mu L_1 \ldots L_{n-1}} F^{\mu L_1 \ldots L_{n-1}} + \frac{4}{D-2} \left(\frac{n-1}{n} \right) F^2 = \frac{4}{D-2} \left(\frac{n-1}{n} \right) F^2, \tag{10}$$

$$T_{00} = -\frac{1}{2n} g_{00} F^2. \tag{11}$$

From the positivity of the energy density, we have $F^2 \geq 0$, and thus $\tilde{T} \geq 0$ in this case.

In the first case, as

$$F_{\mu L_1 \ldots L_{n-1}} F^{\mu L_1 \ldots L_{n-1}} = \frac{4}{n} F^2, \tag{12}$$

we have

$$\tilde{T} = \frac{4}{n} \frac{n-D+1}{D-2} F^2, \tag{13}$$

$$T_{00} = \frac{1}{2n} g_{00} F^2, \tag{14}$$

and with the same logic $F^2 \geq 0$, $D-1 \geq n$, we have again $\tilde{T} \geq 0$. We note that $n = D$ corresponds to a scalar field strength.