A Brief Talk on small-scale cosmology to HEP-TH group in Hongo

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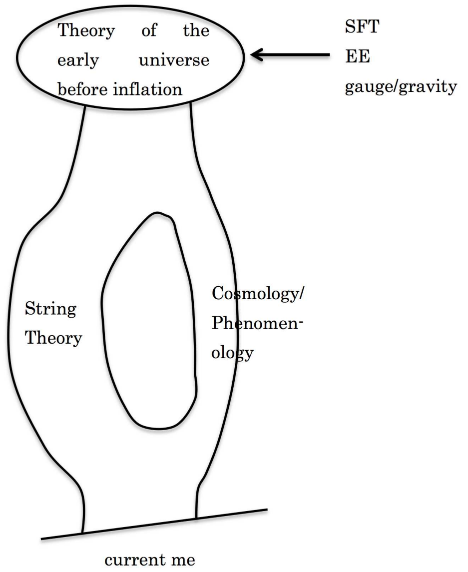
1 Introduction



Today, I am going to focus on my works on the cosmology side. (I still feel like that I am not prepared enough to give a talk on mathematics yet, but I will be back with that kind of topic about two years later) This talk is mainly base on some unpublished work of D. Spergel and myself, and work with A. Natarajan, N. Yoshida [1].

2 Small Scale Fluctuations in the Universe

As what can be seen from figure 1, [2] the current cosmological observation only gives restriction to the shape of the power spectrum up to several Mpc^{-1}. It is natural to think whether the power spectrum will deviate from the standard picture $P_{prid}(k) \propto k^{n_s}$, where $n_s = 0.9608 \pm 0.0054$. [3] As

[1] A. Natarajan, N.Z. & N. Yoshida, Probing the Small Scale Matter Power Spectrum through Dark Matter Annihilation in the Early Universe, arXiv: 1503.03480.

[2] R. HLOZEK et al. THE ATACAMA COSMOLOGY TELESCOPE: A MEASUREMENT OF THE PRIMORDIAL POWER SPECTRUM, arXiv: 1105.4887.

[3] Planck Collaboration, Planck 2013 results. XVI. Cosmological parameters, arXiv: 1303.5076.

we can imagine, it is very difficult to give direct constraint on the small scale fluctuations of the power specturm, while they can be related to a non-conventional initial condition from the unknown high-energy physics[4]. There are also proposals[5] that 21cm observation can help us to understand this region directly, however this kind of observation is extremely difficult, and is not currently helpful.

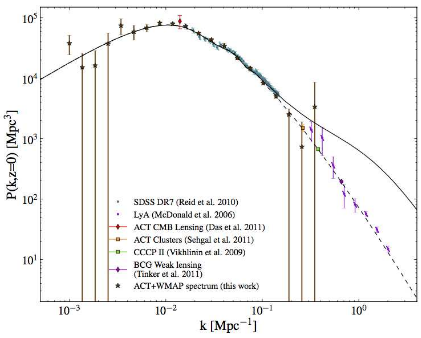


Figure 1: The current obeservational result of the power spectrum

However, a small hill-like deviation from the standard picture in the small-scale region does not affect the cosmology. To impose some constraint on the samll-scale power, we need to consider a stronger power that can result in some observable phenomenon.

$$P_{prid} \propto \begin{cases} k^{n_s} & k < k_p \\ k^{m_s} & k > k_p \end{cases} \tag{1}$$

k_p is the so-called pivot point, where the power changes. A naive explanation of this kind of power spectrum is from some SSB during the inflation, and certainly the realistic power should be connected smoothly between these two phases, but again a small pulse-like deviation has no observable effects. k_p should correspond to the SSB scale if it is the scenario, while the specific value of the SSB scale is model-depending. Our goal is to find the upper bound for the m_s as a function of the pivot point k_p from the observational cosmology. Note that there is no lower bound from the large-scale cosmology as the absence of small scale perturbations induces the same observational results as the standard ΛCDM picture.

3 Restriction from μ-distorsion

In this section, we follow the method proposed by W. Hu et al.(1994)[6] which relates the μ-distorsion and the primordial power analytically.

However, before that, let us go over what the μ-distorsion is first. Before the redshift z_y =

[4]M. Kleban et al. Cosmic 21-cm Fluctuations as a Probe of Fundamental Physics, arXiv: hep-th/0703215.

[5]See the last footnote.

[6]W. Hu et al. POWER SPECTRUM CONSTRAINTS FROM SPECTRAL DISTORTIONS IN THE COSMIC MICROWAVE BACKGROUND, arXiv: astro-ph/9402045.

$2.15 \times 10^4 \Theta_{2.7}^{\frac{1}{2}} (\Omega_B h^2)^{-\frac{1}{2}}$, [7] the equilibrium between photons and electrons is established (We will use $\Omega_B = 0.0482$ in our calculation). Two main processes occur to achieve the equilibrium, one is the photon-number-conserving Compton scattering and another is the double Compton scattering $(e^- + \gamma \to e^- + \gamma + \gamma)$[8]. The former creates a μ distortion from the blackbody radiation,

$$f = \frac{1}{\exp\left(\frac{h\nu}{kT_e} + \mu\right) - 1} \tag{2}$$

and the latter process relieves it. Assuming double Compton scattering is the only way to create photons, we obtain

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} \simeq -\frac{\mu}{t_{DC}} + 1.4 \frac{Q}{\rho_\gamma} \tag{3}$$

where t_{DC} is the double Compton thermalization time and Q/ρ_γ is the fractional rate of energy injection. The solution to this equation is simply [9]

$$\mu \simeq 1.4 \int_0^{t(z_y)} \mathrm{d}t \frac{Q(t)}{\rho_\gamma} \exp\left[-\left(\frac{z}{z_\mu}\right)^{\frac{5}{2}}\right] \tag{4}$$

where $z_\mu = 4.09 \times 10^5 \left(1 - \frac{Y_p}{2}\right)^{-1} \Theta_{2.7}^{\frac{1}{5}} (\Omega_B h^2)^{-\frac{2}{5}}$. A simple estimation gives [10]

$$\frac{Q(t)}{\rho_\gamma} = -\sum_k \frac{1}{3} \frac{\mathrm{d}\langle \Delta^2(k,t) \rangle}{\mathrm{d}t} \tag{5}$$

which reduces equation (4) into

$$\mu \simeq 0.7 \sum_{\mathbf{k}} \int_{z_y}^{\infty} \mathrm{d}z \Delta_J^2(k) \frac{k^2}{k_D^2 z} e^{-\frac{k^2}{k_D^2}} e^{-\left(\frac{z}{z_\mu}\right)^{\frac{5}{2}}} \tag{6}$$

Here we used

$$\langle \Delta^2(k,t) \rangle = \frac{1}{2} \Delta_J^2(k) \exp\left[-\frac{k^2}{k_D^2(t)}\right]$$

where

$$k_D(z) = 2.34 \times 10^{-5} \Theta_{2.7} \left(1 - \frac{Y_p}{2}\right)^{\frac{1}{2}} (\Omega_B h^2)^{\frac{1}{2}} z^{\frac{3}{2}} \mathrm{Mpc}^{-1}$$

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[7] C. Burigana et al. Formation and evolution of early distortions of the microwave background spectrum: a numerical study, Astron. Astrophys. 246,49-58.

[8] Multiple Compton scattering can also contribute, but its probability of occurence is much smaller, therefore double Compton is dominant in the photon-number-non-conservation progress.

[9] W. Hu & J. Silk, Thermalization and spectral distortions of the cosmic background radiation, Phys. Rev. D, 48,485

[10] W. Hu et al. POWER SPECTRUM CONSTRAINTS FROM SPECTRAL DISTORTIONS IN THE COSMIC MICROWAVE BACKGROUND, arXiv: astro-ph/9402045

These formulae look very involved and one can not read much information out of them. In one word, the damping of small-scale fluctuations before recombination, which occurs in the toghtly coupled photon-baryon viscous fliud inside its Jeans length, leads to the distortion of CMB. By solving the Boltzmann equation in tight coupling limit, we obtain (5) and thus all above equations especially (6).

Using $\Theta_{2.7} \equiv \frac{T_{CMB}}{2.7\mathrm{K}} = 1.00944$ and the primordial mass fraction of helium $Y_p = 0.2482$, we get $z_y = 1.44 \times 10^5$ and $z_\mu = 1.97 \times 10^6$. One great feature of this expression is that thanks to the rapid decay of exponential of z, only the physics between redshift z_y and z_μ contributes, the rather unkown physics like inflation or earlier universe almost matters nothing. For this feature, as k_D is a monotonic increasing function of z and $k_D(z_\mu) = 9170.94$ Mpc^{-1}, the μ distortion can be expected to depend extremely weakly on the cutoff used in the real numerical calculation.

Due to the form of transfer function at radiation dominance,

$$\Delta_J^2(k) = 25a_{eq}^2 \frac{k_{eq}^4}{k^4} AP_{prid}(k) \tag{7}$$

where A is a normalization constant and can be determined with the COBE detection at 10°. We used $\frac{\Delta T}{T}(10°) = 1.12 \times 10^{-5}$ in the calculation and adopted the method mentioned in W. Hu et al.'s paper *POWER...* to determine A. The fraction of radiation in current universe is set as $\Omega_r = 4.15 \times 10^{-5} h^{-2} = 8.42 \times 10^{-5}$, which means we are considering the case that all three generations of neutrinos are massless[11].

The result for the standard picture is $\mu = 8.16 \times 10^{-9}$ which agrees very well with recent work[12]. In this calculation, we dropped the cutoff for simplicity. The current constrain on μ is $\mu < 9 \times 10^{-5}$, [13] it is still very far away from the aim, however we can still constrain the primordial function with this.

(A Technical Comment) Since if we change the primordial function with $m_s > 1$, the justification of replacing k_c simply with ∞ will be worrying, we still employ the cutoff in our calculation for new power spectra. It is not convenient to keep the range of z infinite while cutting k off on some scale, a cutoff on z is inserted with $z_c = 1000z_\mu$. It is reasonable to expect that μ decreases as k_v increases or m_s decreases. Reversing the process to set $\mu = 9 \times 10^{-5}$ and find the maximum possible m_s gives us the constrain on m_s as a function of k_v.

The new project PIXIE may be able to detect the μ distortion at level $\mu \sim 5 \times 10^{-8}$. If the result becomes available, we can further constrain m_s down to smaller values (see Table 1).

Table 1: current constraint and possible constraints in the future

μ	9×10^{-5}	1×10^{-5}	1×10^{-6}	5×10^{-8}
$k_v = 100\mathrm{Mpc}^{-1}$	3.44	2.975	2.447	1.645
$k_v = 300\mathrm{Mpc}^{-1}$	4.14	3.587	2.949	1.960
$k_v = 500\mathrm{Mpc}^{-1}$	4.60	3.988	3.285	2.187

[11] As this value is only used in the period of radiation dominance, we do not need to worry at all given the neutrino mass is sufficiently low.

[12] J. Chluba & R. Sunyaev, The evolution of CMB spectral distortions in the early Universe, arXiv:1109.6552v1

[13] D. Fixsen et al. The Cosmic Microwave Background Spectrum from the Full *COBE FiRAS* Data Set, arXiv: astro-ph/9605054.

4 Possible Observational Consequences

A direct physical consequence from this setting is that when $m_s > n_s$, more densed halo objects are formed in earlier periods. This effect itself is not directly observable, but it may lead to some visible phenomena. The most intersting one is that the birth of the first star may become earlier[14]. This kind of event can be inferred from future CMB observations. Another possible effect is that since higher-density halos are formed, a few of them might survive from the tidal stripping and raise the expected annihilation events observed directly in our milky way. Anyway, these effects are rathr connected to future observations and do not give constraints to the power spectrum.

5 Further Constraint from CMB, if DM=WIMPs

A great progress has been made with this idea. The basic content is that with more densed halos, annihilation between WIMPs becomes more intensed, it injects more energy into the CMB power and distorts it. Therefore, from the newly released data from PLANCK[15] in 2015, a further constraint can be applied to the primordial power.

The full matter power specturm is a result of a compromise of the gravitational instability and the "turnover effect" at horizon size at matter-radiation equality. These effects can be taken into account with a so-called transfer function $T(k)$ and a so-called growth factor $D(z)$. $T(k)$ gives the scale-dependence, while $D(z)$ expresses the scale-independent gravitational growth of perturbations.

$$P(k,z) = T^2(k)\frac{D^2(z)}{D^2(z_{eq})}P_{prid}(k) \tag{8}$$

The growth factor can be written in an approximated analytical form with less than $\sim 2\%$ error for $\Omega_m > 0.1$ as $D(z) = \frac{\mathcal{D}(z)}{1+z}$, where [16]

$$\mathcal{D}(z) = \frac{5\Omega_m(z)}{2}\left[\Omega_m(z)^{\frac{4}{7}} - \Omega_\Lambda(z) + (1+\frac{1}{2}\Omega_m(z))(1+\frac{1}{70}\Omega_\Lambda(z))\right]^{-1} \tag{9}$$

and for sufficiently small k, the transfer function can be crudely fitted after the matter-radiation equality as

$$T^2(k) = \frac{N_8}{(1+\alpha_p k + \beta_p k^2)^2} \tag{10}$$

where N_8 is a normalization constant when $P_{prid} = k^{n_s}$ and $\alpha_p = 8(\Omega_m h^2)^{-1}\text{Mpc} = 53.05$ Mpc, $\beta_p = 4.7(\Omega_m h^2)^{-2}\text{Mpc}^2 = 206.68$ Mpc^2. A better fitting formula with 3% accuracy is given by Eisenstein and Hu[17]. The normalization can be determined with the observed mass fluctuation amplitude $\sigma_8 = 0.826$ at the scale of $8h^{-1}\text{Mpc}^{-1}$, where the window function is selected so that

$$\sigma^2(R) = \int_0^{k_c}\frac{\mathrm{d}k}{k}\Delta^2(k)\left[\frac{3j_1(kR)}{kR}\right]^2 \tag{11}$$

[14] S.Hirano, N.Z., N.Yoshida, EARLY STRUCTURE FORMATION FROM PRIMORDIAL DENSITY FLUCTUATIONS WITH A BLUE POWER SPECTRUM, to appear.

[15] Planck collaboration, arXiv: 1502.01582.

[16] D. Eisenstein, An Analytic Expression for the Growth Function in a Flat Universe with a Cosmological Constant, arXiv: astro-ph/9709054v2.

[17] D. Eisenstein & W. Hu, Power Spectra for Cold Dark Matter and its Variants, arXiv: astro-ph/9710252.

$\Delta^2(k) = \frac{k^3}{2\pi^2}P(k)$ and $j_1(x) = \frac{\sin x - x\cos x}{x^2}$ is the spherical Bessel function. k_c is the free-streaming cutoff, of the order $\frac{2\pi}{5\times10^{-10}(1+z_{eq})}\text{Mpc}^{-1} = 3.88\times10^6\ \text{Mpc}^{-1}$, [18] where $z_{eq} = 2.35\times10^4\Omega_m h^2 - 1 = 3241.65$.

A simple way to convert the power spectrum into physical quantity is assuming the fluctuation is Gaussian and making use of critical fluctuation of spherical collapse δ_c to compute the probability for a dark-matter object to form as

$$\int_{\delta_c}^{\infty} \mathrm{d}\delta_M \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\delta_M^2}{2\sigma^2(M)}\right] = \frac{1}{2}\text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M)}\right)$$

According to the argument of the excursion set formalism, we have to multiply a factor of 2 to this. Noting that the overdensed region can be a part of a larger overdensed region with mass $M' > M$, we conclude

$$f_{coll}(>M|z) = \text{erfc}\left(\frac{\delta_c(z)}{\sqrt{2}\sigma(M)}\right) \tag{12}$$

where we have imposed the growth factor on the critical fluctuation to get

$$\delta_c(z) \simeq \frac{3}{5}\left(\frac{3\pi}{2}\right)^{\frac{2}{3}}(1+z) \simeq 1.686(1+z) \tag{13}$$

Defining $\nu_c \equiv \frac{\delta_c(z)}{\sigma(M)}$, we get the comoving number density

$$\frac{\mathrm{d}n}{\mathrm{d}M} = -\sqrt{\frac{2}{\pi}}\frac{\rho_{CDM}}{M}\frac{\mathrm{d}\ln\sigma}{\mathrm{d}M}\nu_c e^{-\frac{\nu_c^2}{2}} \tag{14}$$

For the window function we are using,

$$\frac{\mathrm{d}n}{\mathrm{d}M}(M,z) = -\sqrt{\frac{2}{\pi}}\frac{1}{4\pi R^2 M\sigma^2}\nu_c e^{-\frac{\nu_c^2}{2}}\int_0^{k_c}\frac{\mathrm{d}k}{k}\Delta^2(k)\frac{3\sin kR - 3kR\cos kR}{k^3R^3}\frac{3k^2R^2\sin kR - 9\sin kR + 9kR\cos kR}{k^3R^4} \tag{15}$$

where R is related with M via $M = \frac{4\pi}{3}\rho_{CDM}R^3$. This is the so-called Press-Schechter Formalism, which we will adopt till the end of our discussion.

Assume WIMPs we are considering have mass m_χ and its annihilation cross section is σ_a. If DM's mass density is denoted as ρ_χ, annihilation will occur for $\rho_\chi\langle\sigma_a v\rangle$ times per unit time, and every time for each DM particle, it will release m_χ of energy out. v is the velocity of DM particles and $\langle\ldots\rangle$ is the thermal expectation value. Thus

$$\frac{\mathrm{d}E}{\mathrm{d}t\mathrm{d}V} = \frac{\langle\sigma_a v\rangle}{m_\chi}\rho_\chi^2$$

express the energy injected out per unit volume and per unit time. Translating this information into one halo, we have

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\langle\sigma_a v\rangle}{m_\chi}\int_0^{r_{200}}\mathrm{d}r 4\pi r^2\rho_{halo}^2(r) \tag{16}$$

[18] S. Hofmann et al. Damping scales of neutralino cold dark matter, arXiv: astro-ph/0104173v2

where r_{200} is the radius inside which the mean DM density is 200 times the cosmological average and is where we define the DM halo's boundary and can be expressed as $M = \frac{4\pi}{3} 200\rho_0(1+z_f(M))^3 r_{200}^3$ with ρ_0 the DM density today, and $z_f(M)$ the forming period of a DM halo with mass M in terms of redshift.

The density profile of a DM halo is assumed to take a Navarro-Frenk-White (NFW)-like form

$$\rho(x) = \frac{\rho_s}{x^\alpha(1+x)^\beta} \tag{17}$$

where $x \equiv r/r_s$ and ρ_s, r_s are constants used to characterize a halo. The well-known NFW profile corresponds to $\alpha = 1$, $\beta = 2$. ρ_s and r_s are usually expressed in terms of the concentration parameter $c_{200} \equiv r_{200}/r_s$ and the halo mass M. Taking everything into account, we obtain

$$\frac{\mathrm{d}E_{halo}}{\mathrm{d}t} = \frac{\langle\sigma_a v\rangle}{m_\chi}\frac{200}{3} M\rho_0(1+z_f(M))^3 f_{conc}(c_{200}) \tag{18}$$

where f_{conc} is defined as

$$f_{conc} = \frac{c_{200}^3 \int_0^{c_{200}} \mathrm{d}x x^{2-2\alpha}(1+x)^{-2\beta}}{\left(\int_0^{c_{200}} \mathrm{d}x x^{2-\alpha}(1+x)^{-\beta}\right)^2} \tag{19}$$

a lower cutoff may be necessary for large α. Parameters c_{200}, α, β are usually determined by numerical simulation results. c_{200} is found to be typically $\sim \mathcal{O}(1)$ and the calculation is found not to depend much on the value of c_{200}. We will set $c_{200} = 5$ from now on.

The only missing piece of this calculation is the information of the formation period of a halo of mass M. This is where the Press-Schechter formalism is used. The formalism just assumes that the probability to find a halo of mass M is proportional to $\exp[-\delta_M^2(z)/2\sigma(M)]$ at redshift z. Therefore, a rough estimation can be given as

$$\left\langle\left[\frac{1+z_f(M)}{1+z}\right]^3\right\rangle \simeq \frac{\int_{x_*}^\infty \mathrm{d}x x^3/x_*^3 e^{-x^2}}{\int_{x_*}^\infty \mathrm{d}x e^{-x^2}}$$

Actually in this estimation, once a halo formed, it would still contribute to the above average, which makes it an underestimation, while for a consequent constraint, it merely makes our conclusion conservative, which is not wrong. Note that the Press-Schechter expression for halo number denstiy is in the unit of comoving volume, thus the energy ejected out from DM halo can be expressed as

$$\begin{aligned}
\frac{\mathrm{d}E}{\mathrm{d}t\mathrm{d}V} &= (1+z)^3 \int_{M_{min}}^\infty \mathrm{d}M \frac{\mathrm{d}n}{\mathrm{d}M}\frac{\mathrm{d}E_{halo}}{\mathrm{d}t} \\
&= \frac{\langle\sigma_a v\rangle}{m_\chi}\frac{200}{3}\rho_0 f_{conc}(c_{200})(1+z)^6 \int_{M_{min}}^\infty \mathrm{d}M\ M\frac{\mathrm{d}n}{\mathrm{d}M}\left\langle\left[\frac{1+z_f(M)}{1+z}\right]^3\right\rangle
\end{aligned}$$

where M_{min} corresponds to the minimum DM halo mass with radius of the free-streaming cutoff scale. However, the above expression is not fully correct, since we assumed all DM particles are somehow included in at least one halo. Thus to make it correct, we have to make use of the fraction of DM particles in halos, namely the filling factor, to include the effect of free DM particles.

$$f_{fill}(z) = \frac{1}{\rho_0}\int_{M_{min}}^\infty \mathrm{d}M\ M\frac{\mathrm{d}N}{\mathrm{d}M} \tag{20}$$

Combining with f_{fill}, we can also define another quantity $\zeta(z)$

$$f_{fill}(z)\zeta(z) \equiv \frac{1}{\rho_0}\int_{M_{min}}^{\infty} \mathrm{d}M\ M\frac{\mathrm{d}N}{\mathrm{d}M}\langle\left[\frac{1+z_f(M)}{1+z}\right]^3\rangle \tag{21}$$

$\zeta(z)$ goes to 1 as the redshift z becomes sufficiently large. Finally, we write down the expression of energy absorbed per atom per unit time at redshift z, by assuming that the released energy got absorbed immediately at high redshift [19],

$$\xi(z) = \frac{f_g}{n_b(z)}\frac{\mathrm{d}E}{\mathrm{d}t\mathrm{d}V} = \frac{f_g\langle\sigma_a v\rangle}{m_\chi}\frac{\rho_{crit}\Omega_\chi^2}{\Omega_b}(1+z)^3\left\{1 - f_{fill}(z) + \frac{200}{3}f_{conc}(c_{200})f_{fill}(z)\zeta(z)\right\} \tag{22}$$

A fraction $\eta_i(x_{ion})$ of this energy is consumed as ionization and the left $\eta_h(x_{ion})$ goes into heating. The ionization fraction $x_{ion}(z)$ and the gas temperature $T(z)$ development follows the following equation [20]

$$\begin{aligned} -(1+z)H(z)\frac{\mathrm{d}x_{ion}(z)}{\mathrm{d}z} &= \mu(1-x_{ion}(z))\eta_{ion}(z)\xi(z) - n(z)x_{ion}^2(z)\alpha(z) \\ -(1+z)H(z)\frac{\mathrm{d}T(z)}{\mathrm{d}z} &= -2T(z)H(z) + \frac{2\eta_h(z)}{3k_B}\xi(z) + \frac{x_{ion}(z)(T_\gamma(z)-T(z))}{t_c} \end{aligned} \tag{23}$$

where $H(z)$ is the Hubble constant at redshift z, $\mu \simeq 0.07\mathrm{eV}^{-1}$ is the inverseof average ionization energy per atom (in the case of 76% hydrogen and 24% helium, neglecting double ionization of helium). α is the case-B recombination co-efficient, T_γ is the CMB temperature, k_B is Boltzmann's constant, and t_c is the Compton cooling time scale $\simeq 1.44\mathrm{Myr} \times (30/(1+z))^4$. The practical calculation was done using RECFAST program[21].

The essential effect of the Thomson scattering between CMB photons and ionized gas causes damping of the temperature anisotropy TT power spectrum, and boosts the large angle EE polarization power spectrum. A characteristic quantity to describe such damping is the optical depth

$$\tau(z_1) = \int \mathrm{d}tc\sigma_T n_e(z) = \int_{z_1} n_e(z)\sigma_T\frac{c}{(1+z)H(z)}\mathrm{d}z$$

As the dominant contribution comes from reionization period ($z \sim \mathcal{O}(10)$) which is matter dominant, we can approximate $H(z) \simeq H_0\sqrt{\Omega_m(1+z)^3}$. Combining all facts, we get

$$\tau(z_1) \simeq \frac{c\sigma_T\rho_{crit}}{H_0\bar{m}}\frac{\Omega_b}{\sqrt{\Omega_m}}\int_{z_1}\mathrm{d}z\sqrt{1+z}x_{ion}(z) \tag{24}$$

where $\bar{m}$ is the averaged mass of atoms in gas. The deviation of $x_{ion}(z)$ from the standard picture at higher redshift contributes more to the optical depth. There are two ways to put these data into constraints on m_s. One is using the data of EE polarization power. As has been mentioned before, the ionization caused by DM annihilation will lead to a damping in TT power, and a boost in EE power, however, the CMB power is determined by two parameters, $\sim A_s\exp(-2\tau)$, we can always

[19] The deviation from this assumption can be taken into the effective value of f_g

[20] A. Natarajan, A closer look at CMB constraints on WIMP dark matter, arXiv: 1201.3939

[21] S. Seager, D. Sasselov & D. Scott, How exactly did the Universe become neutral? & A New Calculation of the Recombination Epoch, arXiv: astro-ph/9912182 & arXiv: astro-ph/9909275,respectively

adjust the amplitude A_s to compensate the damping with the result agreeing with observed data. That is to say, we cannot constrain m_s merely with TT data. Another way is to take advantage of the measurement of gravitational lensing of the CMB, which gives the result of σ_8, and hence can be reinterpreted into A_s. Thus from PLANCK 2015, $\tau = 0.066 \pm 0.012$.

The constraint will be imposed according to the value of $\frac{f_g\langle\sigma_a v\rangle}{m_\chi}$, which can be bounded from the CMB data. A fiducial value $\frac{f_g\langle\sigma_a v\rangle}{m_\chi} = \frac{1}{100}$pb/GeV was used in the calculation[22], the result is for $k_p = 100(1000)h\mathrm{Mpc}^{-1}$, $m_s < 1.43(1.63)$ respectively.

[22] It is said that the PLANCK data excluded WIMP mass below $\sim$ 70GeV, for $f_g\langle\sigma_a v\rangle = 1$pb, refer to http://www.cosmos.esa.int/web/planck/ferrara2014